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TIDAL DISSIPATION IN THE MOON

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Abstract 20194

Dissipation of tidal energy in the moon was calculated under the assumption that it can be represented as due to imperfect elasticity. If the factor 1/Q for dissipation per cycle is assumed to be 1/100 for distortional strain energy and 1/1000 for dilatational strain energy, the heat now being generated in the moon by tides is less than .010 ergs/gm/yr.: i.e., negligible compared to radioactive heating by a chondritic composition. Tidal heating would be comparable to radioactive heating, however, if the semi-major axis of the moon's orbit was one-third as great, so the mechanism does limit the possible history of the moon's orbit. Also, appreciable tidal dissipation would cause thermal stresses and be conducive to convection, since it is much greater in the center than near the surface of the moon, and has a non-uniform distribution in latitude (and longitude as well, if rotation is synchronous with revolution).

Urey and others [1959] suggested that tidal dissipation may have significantly contributed to heating of the moon's interior in the past. Kopal [1962] has calculated the tidal dissipation in a moon assumed to be a fluid with Newtonian viscosity. However, since tidal distortion is a relatively high frequency phenomenon, it seems a more modest extrapolation from experience to assume that tidal dissipation in the moon occurs as a result of imperfect elasticity with a factor 1/Q for dissipation per cycle comparable to those estimated for the earth's mantle from polar tides, free oscillations, and latitude variation.

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The tidal disturbing function W is:

$$W = \frac{GM^*}{r^*} \sum_{\ell=2}^{\infty} \left(\frac{r}{r^*}\right)^{\ell} P_{\ell}(\cos S) \tag{1}$$

Where P_ℓ is a Legendre Polynomial; r, ϕ, λ are radius, latitude, and longitude in a moon-fixed coordinate system; the asterisked quantities refer to the disturbing body; GM^* is the product of the gravitational constant and the mass; and S is the arc from (ϕ^*,λ^*) to (ϕ,λ). Apply the addition theorem to equation (1):

$$W = \frac{GM^*}{r^*} \sum_{\ell=2}^{\infty} \left(\frac{r}{r^*}\right)^{\ell} \sum_{m=0}^{\ell} \frac{(\ell-m)!}{(\ell+m)!} (2-\delta_{om}) P_{\ell m}(\sin\phi) P_{\ell m}(\sin\phi^*) \left[\cos m\lambda \cos m\lambda^* + \sin m\lambda \sin m\lambda^*\right] \tag{2}$$

Where $P_{\ell m}$ is the Legendre Associated Function and δ_{om} is the Kronecker delta.

To obtain the variation in time of r^*, ϕ^*, λ^*, they must be expressed in terms of the earth's orbit referred to the moon. For the present orbit this is most simply done using the numerical values of coefficients as given by Brown's theory of lunar motion, as has been recently described by Harrison [1963]. Because we wish to investigate the effects of changing the orbital parameters, and because the necessary computer subroutines had already been written for another purpose (to degree of detail superfluous to the present problem), we used a different development [Kaula, 1961] based on the assumption that the orbit can be considered a Keplerian ellipse at a fixed inclination to the moon's equator with secularly moving node and perigee. For the tidal problem, the most significant omissions under this assumption are short-period perturbations of the semi-major axis and the longitude by the sun. For the present orbit, the largest of these terms (those containing h in the arguments of equations (6)-(8) of Harrison [1963]) have a ratio of about 0.2 to the terms arising from ellipticity of the orbit. This ratio would vary directly with variation in the semi-major axis of the lunar orbit, but would stay about the same with variation in the inclination or eccentricity. So it is consistent with the unavoidable crudeness of our estimates of the dissipation factors $1/Q$ to assume a purely elliptic orbit.

Apply the transformation of equations (7)-(28) in Kaula [1961] to $r^{*1-\ell} P_{\ell m}(\sin \phi^*)\,[\cos m\lambda^*, \sin m\lambda^*]$ in equation (2):

$$W = \frac{GM^*}{a^*} \sum_{\ell=2}^{\infty} \left(\frac{r}{a^*}\right)^{\ell} \sum_{m=0}^{\ell} \frac{(\ell-m)!}{(\ell+m)!} (2-\delta_{om}) P_{\ell m}(\sin\phi) \sum_{pq} F_{\ell mp}(i^*)\, G_{\ell pq}(e^*) \cdot$$

$$\cdot \left[\cos m\lambda \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{\ell-m \text{ odd}}^{\ell-m \text{ even}} \left\{ (\ell-2p)\omega^* + (\ell-2p+q)M^* + m(\Omega^*-\theta) \right\} \right. \tag{3}$$

$$\left. + \sin m\lambda \begin{Bmatrix} \sin \\ -\cos \end{Bmatrix}_{\ell-m \text{ odd}}^{\ell-m \text{ even}} \left\{ (\ell-2p)\omega^* + (\ell-2p+q)M^* + m(\Omega^*-\theta) \right\} \right]$$

Where a^*, e^*, i^*, Ω^*, ω^*, M^* are the Keplerian elements of the earth's orbit referred to the moon's equator and a departure point thereon fixed with respect to inertial space; $F_{\ell mp}(i^*)$ and $G_{\ell pq}(e^*)$ are polynomials of the sine and cosine of the inclination and of the eccentricity, respectively, and θ is the "lunar sidereal time": the angle between the inertially fixed departure point and the point on the moon from which selenographic longitudes are measured. We assume the rate $\dot{\theta}$ to be constant, which is equivalent to neglecting the physical libration.

We abbreviate equation (3) as:

$$W=\sum_{l=2}^{\infty} r^{l} \sum_{m=0}^{l} P_{lm}(\sin\phi) \sum_{g} a_{lmg} \Big[\cos m\lambda \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} (\sigma_{lmg} t - t_{lmg}) + \sin m\lambda \begin{Bmatrix} \sin \\ -\cos \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} (\sigma_{lmg} t - t_{lmg}) \Big] \tag{4}$$

Where the single subscript g replaces the subscript pair p,q; the amplitude

$$a_{lmg} = \frac{GM^*}{a^*} \left(\frac{r}{a^*}\right)^{l} \frac{(l-m)!}{(l+m)!} (2-\delta_{om}) F_{lmp}(i^*)\, G_{lpq}(e^*); \tag{5}$$

and the rate

$$\sigma_{lmg} = (l-2p)\dot{\omega}^* + (l-2p+q)\dot{M}^* + m(\dot{\Omega}^* - \dot{\theta}) \tag{6}$$

The strain energy per unit volume, divided into shear (or distortional) and compressive (or dilatational) terms [p.12, Jeffreys, 1959]:

$$E = \mu e'_{ij} e'_{ij} + \frac{k}{6} e_{mm} e_{mm} \tag{7}$$

summing over repeated subscripts. In equation (7), μ is the rigidity, k is the bulk modulus, e_{ij} is the strain tensor and e'_{ij} is the part of the strain tensor expressing departures from symmetry:

$$e'_{ij} = e_{ij} - \frac{1}{3}\delta_{ij}\, e_{kk} \tag{8}$$

where δ_{ij} is the Kronecker delta.

If the tidal disturbing function is expressed as a sum of spherical harmonics:

$$W = \sum_{l=2}^{\infty} r^l \sum_{m=0}^{l} P_{lm}(\sin\phi)\left[A_{lm}(t)\cos m\lambda + B_{lm}(t)\sin m\lambda\right] \tag{9}$$

the strain tensor e_{ij} at any point (r,ϕ,λ) can be expressed as

$$e_{ij} = \sum_{l,m}\left[A_{lm}(t)\,\epsilon_{ijlmc}(r,\phi,\lambda) + B_{lm}(t)\,\epsilon_{ijlms}(r,\phi,\lambda)\right] \tag{10}$$

where $\epsilon_{ijlmc,s}$ is the response of a planetary model of specified shear modulus, bulk modulus, and density to a unit coefficient in the tidal disturbing function.

Comparing equations (4) and (9), we have:

$$A_{lm}(t) = \sum_g a_{lmg} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} (\sigma_{lmg} t - t_{lmg}) \tag{11}$$

$$B_{lm}(t) = \sum_g a_{lmg} \begin{Bmatrix} \sin \\ -\cos \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} (\sigma_{lmg} t - t_{lmg}) \tag{12}$$

Substituting equations (10), (11), (12) in equation (7) and sorting out the algebra which results obtains:

$$E = \sum_{l,m,g,u,v,h} \Big[\left(S_{1_{lmguvh}} + C_{1_{lmguvh}}\right) \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{b \text{ odd}}^{b \text{ even}} \left\{ (\sigma_{lmg} + \sigma_{uvh}) t - (t_{lmg} + t_{uvh}) \right\}$$

$$+ \left(S_{2_{lmguvh}} + C_{2_{lmguvh}}\right) \begin{Bmatrix} \sin \\ \cos \end{Bmatrix}_{b \text{ odd}}^{b \text{ even}} \left\{ (\sigma_{lmg} + \sigma_{uvh}) t - (t_{lmg} + t_{uvh}) \right\}$$

$$+ \left(S_{3_{lmguvh}} + C_{3_{lmguvh}}\right) \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{b \text{ odd}}^{b \text{ even}} \left\{ (\sigma_{lmg} - \sigma_{uvh}) t - (t_{lmg} - t_{uvh}) \right\} +$$

$$+\left(S_{4lmguvh}+C_{4lmguvh}\right)\begin{Bmatrix}-\sin\\ \cos\end{Bmatrix}_{b\ \text{odd}}^{b\ \text{even}}\left\{\left(\sigma_{lmg}-\sigma_{uvh}\right)t-\left(t_{lmg}-t_{uvh}\right)\right\}\Bigg] \qquad (13)$$

Where $b=l+u-m-v$; (u,v) summations start at (l,m); and

$$S_{1lmguvh}=a_{lmg}a_{uvh}\frac{\mu}{2}\left(\epsilon'_{ijlmc}\epsilon'_{ijuvc}-\epsilon'_{ijlms}\epsilon'_{ijuvs}\right)\left(2-\delta_{uv}^{lm}\right)$$

$$C_{1lmguvh}=a_{lmg}a_{uvh}\frac{k}{12}\left(\epsilon_{iilmc}\epsilon_{jjuvc}-\epsilon_{iilms}\epsilon_{jjuvs}\right)\left(2-\delta_{uv}^{lm}\right)$$

$$S_{2lmguvh}=a_{lmg}a_{uvh}\frac{\mu}{2}\left(\epsilon'_{ijlmc}\epsilon'_{ijuvs}+\epsilon'_{ijlms}\epsilon'_{ijuvc}\right)\left(2-\delta_{uv}^{lm}\right)\frac{(-1)^{u-v}+(-1)^{l-m}}{2}$$

$$C_{2lmguvh}=a_{lmg}a_{uvh}\frac{k}{12}\left(\epsilon_{iilmc}\epsilon_{jjuvs}+\epsilon_{iilms}\epsilon_{jjuvc}\right)\left(2-\delta_{uv}^{lm}\right)\frac{(-1)^{u-v}+(-1)^{l-m}}{2}$$

$$S_{3lmguvh}=a_{lmg}a_{uvh}\frac{\mu}{2}\left(\epsilon'_{ijlmc}\epsilon'_{ijuvc}+\epsilon'_{ijlms}\epsilon'_{ijuvs}\right)\left(2-\delta_{uv}^{lm}\right)(-1)^{u-v}$$

$$C_{3lmguvh}=a_{lmg}a_{uvh}\frac{k}{12}\left(\epsilon_{iilmc}\epsilon_{jjuvc}+\epsilon_{iilms}\epsilon_{jjuvs}\right)\left(2-\delta_{uv}^{lm}\right)(-1)^{u-v}$$

$$S_{4\ell mguvh} \simeq a_{\ell mg} a_{uvh} \frac{\mu}{2} \left(\epsilon'_{ij\ell mc} \epsilon'_{ijuvs} - \epsilon'_{ij\ell ms} \epsilon'_{ijuvc} \right) \left(2 - \delta_{uv}^{\ell m} \right) \frac{1-(-1)^b}{2}$$

$$C_{4\ell mguvh} = a_{\ell mg} a_{uvh} \frac{k}{12} \left(\epsilon_{ii\ell mc} \epsilon_{jjuvs} - \epsilon_{ii\ell ms} \epsilon_{jjuvc} \right) \left(2 - \delta_{uv}^{\ell m} \right) \frac{1-(-1)^b}{2} \quad (14)$$

For any particular term of subscripts ℓ,m,u,v,g,h, the energy dissipated in one cycle of duration $2\pi/(\sigma_{\ell mg} + \sigma_{uvh})$ will be:

$$\Delta E_{\ell mguvh+} = 2\pi \left[\left| S_{1\ell mguvh}/Q_s + C_{1\ell mguvh}/Q_c \right| + \left| S_{2\ell mguvh}/Q_s + C_{2\ell mguvh}/Q_c \right| \right] \quad (15)$$

and in one cycle of duration $2\pi/(\sigma_{\ell mg} - \sigma_{uvh})$:

$$\Delta E_{\ell mguvh-} = 2\pi \left[\left| S_{3\ell mguvh}/Q_s + C_{3\ell mguvh}/Q_c \right| + \left| S_{4\ell mguvh}/Q_s + C_{4\ell mguvh}/Q_c \right| \right] \quad (16)$$

To obtain the contributions to energy dissipation per unit time, multiply equations (15) and (16) by the absolute values of the rates per unit time, $|\sigma_{\ell mg} + \sigma_{uvh}|$ and $|\sigma_{\ell mg} - \sigma_{uvh}|$. Sum over all subscripts to obtain the mean dissipation rate at a particular point fixed in the moon:

$$\dot{E}_d(r,\phi,\lambda) = \sum_{\ell,m,g,u,v,h} \left[\Delta E_{\ell mguvh+} \left| \sigma_{\ell mg} + \sigma_{uvh} \right| + \Delta E_{\ell mguvh-} \left| \sigma_{\ell mg} - \sigma_{uvh} \right| \right] \tag{17}$$

If the rotation is synchronous with revolution, as exists at present,

$$\dot{\omega}^* + \dot{M}^* + \dot{\Omega}^* - \dot{\theta} = 0, \tag{18}$$

there are terms which will contribute only through their amplitude $a_{\ell mg}$ but not through their rate $\sigma_{\ell mg}$. In the synchronous case, a reference longitude must be fixed. Making this reference longitude the mean direction of the earth sets zero all terms containing $\sin\{m(\omega^* + M^* + \Omega^* - \theta)\}$ and sets unity all terms containing $\cos\{m(\bar{\omega}^* + \bar{M}^* + \bar{\Omega}^* - \bar{\theta})\}$: i.e., in equation (10) the contribution to $A_{\ell m}(t)$ will be $a_{\ell mg}$ for ℓ-m even and 0 for ℓ-m odd, and to $B_{\ell m}(t)$ it will be 0 for ℓ-m even and $-a_{\ell mg}$ for ℓ-m odd. Practically the only term for which this effect is significant is $(\ell,m,p,q) = (2,2,0,0)$.

In addition, there will be degeneracies for m = 0 in all cases and for m ≠ 0 in the synchronous case, requiring the combination of terms before proceeding as in equations (13)-(16). In these cases, the rate for the term of subscripts (ℓ,m,p,q) will be the negative of the rate for the term of subscript (ℓ,m, ℓ-m-p, -q). If the rate is taken as that of the (ℓ,m,p,q) term, then the amplitude for the cosine coefficient will be:

$$a_{\ell mg} = a_{\ell mh} + (-1)^{\ell - m} a_{\ell mi} \qquad (19)$$

and for the sine coefficient:

$$b_{\ell mg} = a_{\ell mh} - (-1)^{\ell - m} a_{\ell mi} \qquad (20)$$

where the subscript correspondence is h with (p,q) and i with (ℓ-m-p, -q), and $a_{\ell mh}$, $a_{\ell mi}$ are computed by equation (5). Then equation (14) must be modified so that $b_{\ell mg}$, b_{uvh} coefficients appear in front of $\epsilon_{ij\ell ms}$, ϵ_{ijuvs}, etc. terms in place of $a_{\ell mg}$, a_{uvh}.

Another set of degeneracies occuring in the synchronous case arises because terms of subscript (ℓ, m, p, q) will have rates equal to terms of subscript (ℓ, m+2i, p-i, q), where i is any integer.

Including terms for which the disturbing function rate is zero in effect makes the energy dissipation rate a function of the constant value of the strain, which raises the question of whether strains from other than tidal causes should be considered as well. Since we are interested in the dissipation over geological durations of time, these terms perhaps should be omitted because in such time we would expect non-oscillating strains to be removed by anelastic processes. But then the dissipation rate obtained would be an absolute minimum for the assumed Q. The moments of inertia of the moon indicate that it now contains strains larger than tidal, so leaving in the non-oscillating tidal terms should yield a dissipation rate unlikely to be too high.

The quantities $a_{\ell mg}$, $\sigma_{\ell mg}$; $\ell = 2$, $m = 0,1,2$ where calculated from equations (5) and (6), for a variety of lunar orbits, using subroutines for $F_{\ell mp}(i^*)$ and $G_{\ell pq}(e^*)$ originally devised for analysis of close satellite orbits, and computing the rates $\dot{\omega}^*$, $\dot{M}^*$, $\dot{\Omega}^*$ by the methods described in Kaula [1961].

The strain tensors $\epsilon'_{ij\ell m}$, ϵ_{iilm} were calculated using the formulation of the earth tide problem of Alterman and others [1959], which has also been used by Takeuchi and others [1962] and Longman [1963]. In this formulation, the basic variables are the radial factors of vector spherical harmonic expressions of the displacements, stresses, and potential terms: y_1, of the radial displacement; y_2, of the compressive stress; y_3, of the tangential displacement; y_4, of the shear stress; y_5, of the potential; y_6, of the potential gradient less the contribution thereto of the radial displacement. The equations of equilibrium then become a system of six first order equations:

$$\frac{dy_i}{dr} = P_{ij}\, y_j \tag{21}$$

The P_{ij} are functions of r, k, μ, g, and ρ. Three columns of P_{ij} have terms of $O(r^{-2})$, so the requirement of regularity at the origin eliminates three constants of integration. The three surface conditions of zero tangential stress, zero radial stress, and the potential gradient being related to the poetntial as a spherical harmonic in free space make the problem determinate.

After solving equation (21) numerically for the y_i's corresponding to a particular harmonic $Y_{\ell m} = r^{\ell} S_{\ell m}$, the contribution to the strain matrix ϵ_{ij} is calculated by (here θ is colatitude, ϕ is longitude, and λ is $k - \frac{2}{3}\mu$):

$$
\begin{aligned}
\epsilon_{\theta\theta} &= 2\frac{S_{\ell m}}{r} y_1 + \frac{2}{r}\frac{\partial^2 S_{\ell m}}{\partial \theta^2} y_3 \\
\epsilon_{\phi\phi} &= 2\frac{S_{\ell m}}{r} y_1 + \frac{2}{r\sin\theta}\left(\frac{1}{\sin\theta}\frac{\partial^2 S_{\ell m}}{\partial \phi^2} + \cos\theta\frac{\partial S_{\ell m}}{\partial \theta}\right) y_3 \\
\epsilon_{rr} &= -\frac{4\lambda}{\lambda+2\mu}\cdot\frac{S_{\ell m}}{r} y_1 + \frac{2\ell(\ell+1)\lambda}{\lambda+2\mu}\cdot\frac{S_{\ell m}}{r} y_3 + \frac{2S_{\ell m}}{\lambda+2\mu} y_2 \\
\epsilon_{\theta\phi} &= \frac{2}{r\sin\theta}\left(\frac{\partial^2 S_{\ell m}}{\partial\theta\partial\phi} - \cot\theta\frac{\partial S_{\ell m}}{\partial\phi}\right) y_2 \\
\epsilon_{\phi r} &= \frac{1}{\mu\sin\theta}\frac{\partial S_{\ell m}}{\partial\phi} y_4 \\
\epsilon_{r\theta} &= \frac{1}{\mu}\frac{\partial S_{\ell m}}{\partial\theta} y_4
\end{aligned}
\qquad (22)
$$

Equation (22) can be derived using the equations on page 56 of Love [1927], applying a factor of 1/2 to the off diagonal components to be consistent with tensor convention. The $\epsilon_{ij\ell m}$'s from the $Y_{\ell m}$'s were used in equation (14).

The pole tide suggests a Q of more than 100 for the earth at a 14-month period [p.162, Munk & MacDonald, 1960], while the latitude variations indicate a Q of about 40 at the same period [p.148, Munk & MacDonald, 1960; pp.255-259, Jeffreys, 1959]. The free oscillations of the earth indicate Q's of 170 to 400 for periods less than an hour [Benioff and others, 1961; Ness and others, 1961], except for a Q of 7500 for the $S_0^{\,0}$, the only mode which is purely compressive [Ness and others, 1961]. Models for rock creep proposed by Jeffreys and Crampin [1960], ~~and~~ J. R. MacDonald [1961] and Lomnitz [1962] suggest Q's between 60 and 100 for semi-monthly and monthly periods. A shear Q_s of 100 thus seems a reasonable compromise. Considering that excitation from the atmosphere and oceans may maintain the $S_0^{\,0}$ free oscillation [Ness and others, 1961], we assume a compressive Q_c of 1000.

The strain energies were calculated for several lunar models proposed by Harrison [1963]. However, since the uncertainty in Q reduces this problem to one of estimating order of magnitude, this discussion will be limited to a homogeneous moon of density 3.34 gms cm^{-3}, rigidity 7.38×10^{11} dynes cm^{-2}, and bulk modulus 1.23×10^{12} dynes cm^{-2}. The Love numbers obtained for this model by the numerical solution of equation (15) were .0344 for h, .0195 for k, and .0095 for ℓ.

The thermal histories of lunar models with chondritic composition have been calculated by G. J. F. MacDonald [1959]. Even with a cold origin, these models come very close to melting at depths in the moon exceeding 500 km. The chondritic composition used had radioactive contents of 8.0×10^{-4}, 1.1×10^{-8}, and 4.4×10^{-8} g/g for potassium, uranium, and thorium, respectively, which yield a thermal energy output of 1.59 ergs/g/year at present and 12.8 ergs/g/year 4.5×10^{9} years ago. Hence for tidal dissipation to be significant, it should contribute on the order of 5 ergs/g/year, or .04 ergs/cm^3/day.

The results for the present orbit of the moon are shown in Figure 1 in the form of maps of the energy dissipation at four levels within the moon. Since the dissipation is symmetric about the equator and two meridians at right angles, we need show only one octant for each level. The evident features are firstly that the tidal dissipation is at present a negligible source of heat, and secondly that the distribution of the heating is extremely non-uniform both radially and laterally. This variability of distribution suggests that if the moon's orbit had ever been such that the overall heating was appreciable, large thermal stresses would have resulted, leading to convection or some other form of mass motion.

The amplitudes and rates were therefore calculated for different orbital specifications. In turn each element was varied, holding the others fixed at the present values: 1) the semi-major axis (holding the rotation synchronous with revolution about the earth); 2) the eccentricity; 3) the inclination; and 4) the rotation rate. The results are displayed in Figure 2, in the form of curves showing variation in the average ratio to the present dissipation with variation in the orbital elements. The variation is particularly marked with variation in the semi-major axis. If the semi-major axis were only one-third as great as it is now, the criterion of .04 ergs/cm^3/day would be exceeded for most of the moon. At the secular acceleration calculated by Munk & MacDonald [1960], the moon would have been at this distance about 10^9 years ago.

The pattern of energy dissipation shown in Figure 1 is composed of even degree harmonics symmetric about the equator: (4,4), (4,2), (4,0), (2,2), and (2,0). In a moon close enough that heating by tidal dissipation was large enough to cause convection, the 2d degree terms in its mass distribution would be determined mainly by the gravitational attraction of the earth. However, the fourth degree terms would be determined by the convective pattern. It therefore will be of interest when variations in the gravitational field and external form of the moon are better determined, to find out whether these fourth degree terms are markedly larger than other terms, such as the third degree terms. If they are, it would be a strong indication that the moon was once close enough for heating by tidal dissipation to cause convection, and hence would lend further evidence as to the moon's origin.

In conclusion, we can say that heating by tidal dissipation is currently insignificant (unless Q factors from the earth's mantle are wrong by a factor of about 10^2), but that this study confirms the suggestion of Urey and others [1959] that it may have been important in the past when the moon came much closer to the earth.

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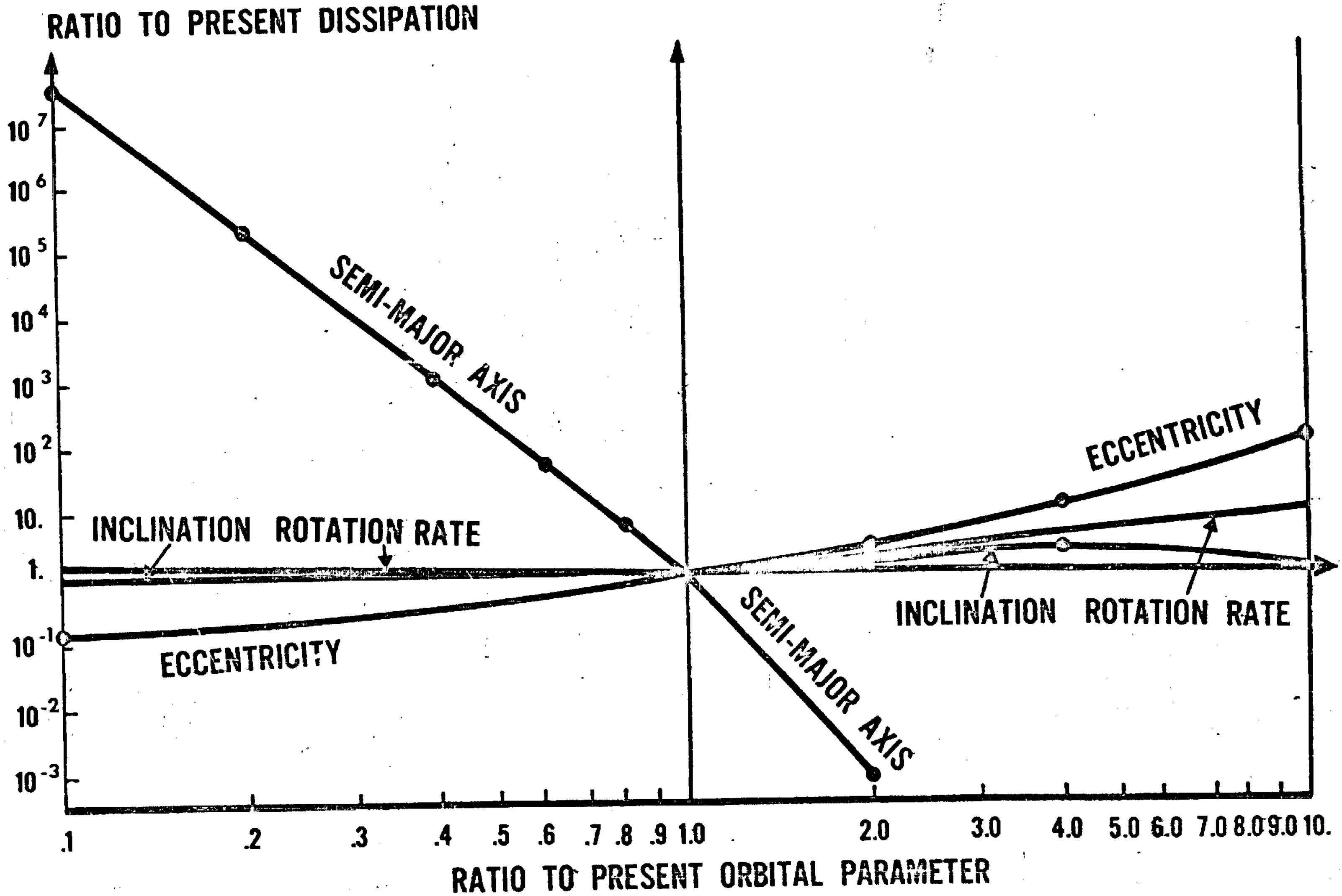
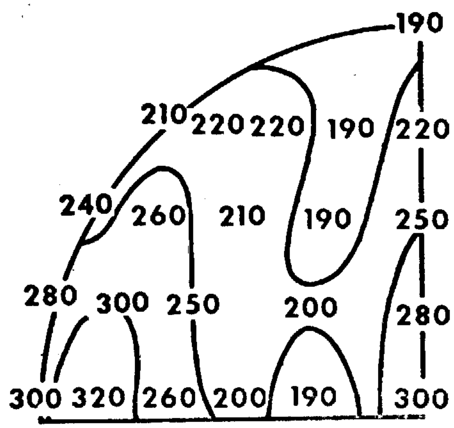
VARIATION OF TIDAL ENERGY DISSIPATION IN THE MOON WITH ONE-AT-A-TIME VARIATION IN ORBITAL PARAMETERS



TIDAL ENERGY DISSIPATION IN THE MOON



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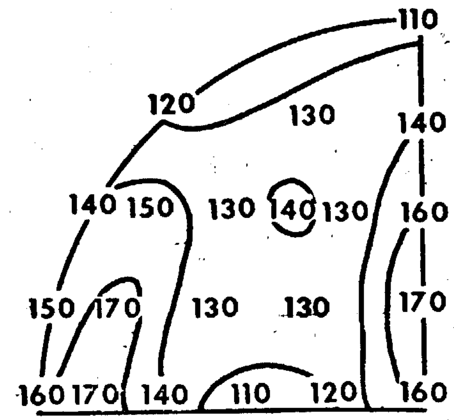

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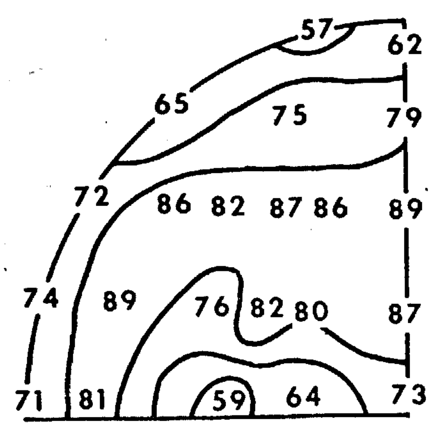

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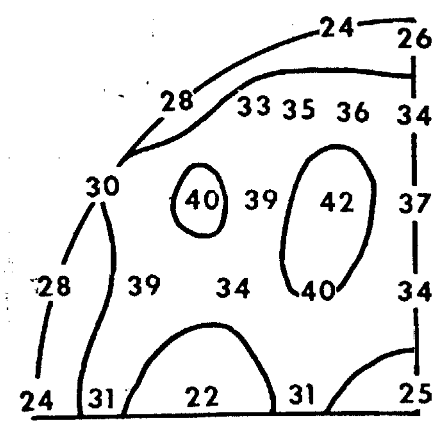

R=.97 LUNAR RADII

UNITS: 10^{-6} ERGS/ CM^3/ DAY